MATH 290 Qualifying Exam syllabus

1 290A: Theory of ODEs and Perturbation Methods

Reading List:

The two bolded references (Perko, Holmes) are the primary references for the course.

Theory of ODEs:

- C. Chicone. *Ordinary differential equations with applications.* Springer, 2006.
- J. Guckenheimer, P. Holmes. *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields.* Springer, 1983.
- J. Meiss. *Differential Dynamical Systems.* SIAM, 2017.
- **L. Perko. *Differential equations and dynamical systems.* Springer, 2001.**

Perturbation methods:

- C. Bender, S. Orszag. *Advanced Methods for Scientists and Engineers: Asymptotic Methods and Perturbation Theory.* Springer, 1999.
- E.J. Hinch. *Perturbation Methods.* Cambridge University Press, 1991.
- C. Kuehn. *Multiple Time Scale Dynamics.* Springer, 2015.
- **M. Holmes. *Introduction to perturbation methods.* Springer, 2013.**

Sample weekly schedule of topics:

- Week 1: existence/uniqueness theory, smooth dependence on initial conditions/parameters
  [Perko, sections 2.1–2.4]
- Week 2: flows, orbits, invariant sets, α/ω limit sets, Poincaré–Bendixon
  [Perko, sections 2.5, 3.2, 3.7]
- Week 3: linear theory, fundamental matrix solution, matrix exponential
  [Perko, sections 1.4, 1.8–1.10]

- Week 4: nonlinear theory near hyperbolic fixed points, Hartman–Grobman theorem, stable manifold theorem

  [Perko, sections 2.6–2.8]

- Week 5: center manifold theorem, local bifurcations of fixed points (fold, pitchfork, transcritical, Hopf)

  [Perko, sections 2.12, 4.2, 4.4]

- Week 6: Poincaré map, Periodic coefficients, Floquet theory

  [Perko, section 3.3–3.5]

- Week 7: intro to perturbations of ODEs

  [Holmes, sections 1.4, 1.6]

- Week 8: singularly perturbed ODEs, boundary layers and matched asymptotic expansions

  [Holmes, sections 2.1–2.3]

- Week 9: multiple scales expansions

  [Holmes, sections 3.1–3.3]

- Week 10: leftover topics & final exam review

2 290B: Numerical Ordinary Differential Equations and Numerical Linear Algebra

Textbooks:

- Numerical Analysis, 2nd edition. W. Gautschi. Birkhauser (2012)
- Applied Numerical Linear Algebra, J. Demmel. SIAM (1997).

Additional background sources:

- Numerical Analysis, R. Scott. 2nd edition (2016).

  https://people.cs.uchicago.edu/ ridg/newna/natwo.pdf

- Classical numerical analysis: A comprehensive course, A. Salgado & S. Wise, Cambridge (2023).

Course Description

This course develops and analyzes numerical methods for two fundamental problems in applied mathematics: the numerical solution of ordinary differential equations, and the application of numerical methods to problems in linear algebra. The course includes both theory and numerical implementation of the algorithms.

List of Topics:

1. Initial value problems for ordinary differential equations
   - One step methods, error monitoring, adaptive methods, stiff systems (Gautschi, Chapter 5)
   - Multistep methods, error monitoring, analytic theory of order and stability (Gautschi, Chapter 6)
2. Numerical linear algebra
   - Linear least squares problems. Normal equations, QR decomposition, Householder/Givens rotations, Rank deficient least squares (Demmel, Chapter 3)
   - Eigenvalue problems
     - Nonsymmetric eigvenvalue problems. Power method, inverse iteration, QR iteration, Hessenberg reduction, QR iteration with implicit shifts (Demmel, Chapter 4)
     - Symmetric Eigenvalue problems. QR Iteration. (Demmel, Chapter 5)

3 290C: Calculus of Variations

Reading List:

The two bolded references (Perko, Holmes) are the primary references for the course.

Course Description

Calculus of Variations deals with optimization problems where the variables, instead of being finite-dimensional as in ordinary calculus, are functions. This course treats the foundations of calculus of variations and gives examples of some (classical and modern) applications within physics and engineering science.

Textbook and References

Pdf files of lectures will be distributed in class. However, the following books are good references for reading:

- *Lecture Notes on Calculus of Variations* by Kung Ching Chang https://www.worldscientific.com/worldscibooks/10.1142/10157
- Chapter 8: Calculus of Variation in *Partial Differential Equations (2nd Edition)* by Lawrence C. Evans https://www.amazon.com/Partial-Differential-Equations-Graduate-Mathematics/dp/0821849743
- *Functional analysis, calculus of variations and optimal control.* by F. Clarke, Springer https://link.springer.com/book/10.1007/978-1-4471-4820-3

Sample Weekly Schedule of Topics

- Week 1: Introduction to Calculus of Variation
- Week 2: Classic Theory in 1D
- Week 3: Brief Review of Functional Analysis
- Week 4: Sobolev Spaces (Chapter 10 in K.C. Chang)
- Week 5: Existence of Global Minimum (Chapter 8 in Evans and Chapter 9 in K.C. Chang)
- Week 6: Brief Review of Convex Analysis (Chapter 4 in Clarke)
- Week 6-7: Hamilton-Jacobi Theory (Chapter 5 in K.C. Chang)
- Week 8-9: Constraint Problems (Chapter 7 in K.C. Chang)
- Week 10: Noether Theorem (Chapter 8 in K.C. Chang)
- Week 11: Final exam.

These are tentative weekly schedules. Links to previous course notes and references are provided.